A Physically-Based Electron Mobility Model for Silicon Device Simulation

G. Kaiblinger-Grujin, T. Grasser, and S. Selberherr

Institute for Microelectronics, TU Vienna
Gusshausstr. 27–29, A-1040 Vienna, Austria

Abstract

We present an analytical low-field electron mobility formula for silicon which treats the dependence on all common dopants, such as P, As, Sb, and B in a unified manner. The expressions are derived from Monte-Carlo (MC) calculations which are based on a theoretical approach to ionized impurity scattering that inherently distinguishes the dopant species. From these first principle data we derive analytical expressions for the majority and minority mobility valid in the temperature range (70–500 K) and up to an impurity concentration of 10^{22} cm^{-3}. The agreement with experimental data is excellent. Not only the lower majority electron mobility in As- and Sb-doped Si, but also the higher minority electron mobility in B-doped Si compared to the majority mobility is confirmed. Hence, this universally usable mobility model is very well suited for device simulation purposes.

1. Introduction

The electron mobility in silicon is an important parameter for device design and analysis. Accurate mobility models are necessary for predictive simulation due to the direct dependence of the current on mobility, which is often the most desired quantity. It is well known that under low fields the mobility depends on the doping concentration and on temperature. However, the mobility values in n-type Si may differ by more than 30% even at room temperature depending on the donor species [1]. To simulate bipolar transistors it becomes necessary to distinguish between majority and minority mobility values. Experiments have shown that the minority electron mobility may exceed the majority electron mobility by a factor of 16 at an acceptor concentration of about $6 \cdot 10^{19}$ cm^{-3} [2].

2. Physical Model

Our theoretical model combines the classic atomic model of Thomas-Fermi with a variational principle to obtain a unique charge density distribution for each dopant [3]. With this characteristic charge density we derived scattering rates for each dopant useful for a Monte-Carlo simulator to finally obtain the low field electron mobility.

As the experimental mobility data are reproduced with high accuracy, the computed mobility data can be used as the basis for deriving fit formulae which are useful for a conventional device simulator.

3. Majority Electron Mobility

The dopands are distinguished by their atomic number Z. The electron mobility in n-Si can be fitted by

$$\mu_{n,D}(N_D, T, Z_D) = \frac{\mu_0 - g - h}{1 + \left(\frac{N_D}{C_1}\right)^{\alpha_1}} + \frac{g}{1 + \left(\frac{N_D}{C_2}\right)^{\alpha_2}} + h \tag{1}$$

with temperature-dependent parameters defined as follows:

$$\mu_0(T) \left[\frac{\mathrm{cm}^2}{\mathrm{Vs}}\right] = 380 + 20700\, e^{-\frac{T}{100\,\mathrm{K}}} \tag{2}$$

$$g(T, Z) \left[\frac{\mathrm{cm}^2}{\mathrm{Vs}}\right] = 2\left(9 - 4\frac{Z}{Z_P}\right) + \left(7\frac{Z}{Z_P} + 208\right) e^{-\frac{T}{200\,\mathrm{K}}} \tag{3}$$

$$h(T, Z) \left[\frac{\mathrm{cm}^2}{\mathrm{Vs}}\right] = \frac{9 - \frac{Z}{Z_P}}{\left(\frac{T}{300\,\mathrm{K}}\right)} \tag{4}$$

$$\alpha(T) = 0.9 - 0.18\left(\frac{T}{300\,\mathrm{K}}\right) \tag{5}$$

$$\beta(T) = 0.46 + 1.05\left(\frac{T}{300\,\mathrm{K}}\right) \tag{6}$$

$$\frac{C_1(T)}{10^{16}}\left[\mathrm{cm}^{-3}\right] = 11.85\left(\frac{T}{300\,\mathrm{K}}\right)^3 + 0.45 \tag{7}$$

$$\frac{C_2(T, Z)}{10^{20}}\left[\mathrm{cm}^{-3}\right] = \left[3 + \left(\frac{Z}{Z_P}\right)^{-2}\right] \cdot \left[1.2 - \left(\frac{T}{300\,\mathrm{K}}\right) e^{\frac{3-7T}{300\,\mathrm{K}}}\right] \tag{8}$$

The coefficients are normalized for 300 K and the most common donor phosphorus with $Z_P = 15$. Note that only three parameters are dopant-dependent through the atomic number Z: g, h, C_2. With $Z = Z_B = 5$ (4) and (8) give also the temperature-dependence of minority electrons (see Section 4). The dopant-dependence of the parameters is evident from (1) as the second rational term becomes dominant with increasing donor concentration where the dopant-dependence is most pronounced. We found that the temperature-dependence of the lattice electron mobility (2) shows an exponential decrease below 150 K rather than a simple power-law fall-off. Fig. 1 and Fig. 3 show the electron mobility as a function of donor concentration for various temperatures. The agreement with available experimental data is excellent for all dopants. Assuming no correlation between different donor species we can apply pragmatically the Matthiessen rule to obtain the effective majority mobility. A more rigorous approach to combine the mobility components can be readily imagined. For the purpose of fit formulas, however, it does not seem to be necessary, in particular having in mind the spread of the experimental data at high doping levels (cf. Fig. 2).

$$\mu_{n,P,As}(N_P, N_{As}) = \left(\frac{1}{\mu_{n,P}(N_P)} + \frac{1}{\mu_{n,As}(N_{As})} - \frac{1}{\mu_0}\right)^{-1} \tag{9}$$

4. Minority Electron Mobility

There exist only few experimental data in limited doping ranges for the electron mobility in B-doped Si (e.g., [2]). Unfortunately, the uncertainties due to the experi-

mental error are rather high, and pronounced discrepancies among the reported data are found. For the electron mobility in B-doped Si we make the ansatz

$$\mu_{n,B}(N_B, T, Z_B) = \frac{\mu_0 + m - k - h}{1 + \left(\frac{N_B}{C_1}\right)^{\alpha_1}} + \frac{k}{1 + \left(\frac{|N_B - C_a|}{C_2}\right)^{\alpha_2}} - \frac{m}{1 + \left(\frac{|N_B - C_b|}{C_3}\right)^{\alpha_3}} + h \quad (10)$$

with the following set of equations together with (4) and (8) with $Z = Z_B = 5$ for the temperature-dependent parameters:

$$\gamma(T) = \frac{0.6}{\left(\frac{T}{300\,\mathrm{K}}\right)} + 1.4 \quad (11)$$

$$k(T)\left[\frac{\mathrm{cm}^2}{\mathrm{Vs}}\right] = \frac{134}{\left(\frac{T}{300\,\mathrm{K}}\right)} + 70 \quad (12)$$

$$m(T)\left[\frac{\mathrm{cm}^2}{\mathrm{Vs}}\right] = \frac{65}{\left(\frac{T}{300\,\mathrm{K}}\right)} + 73 \quad (13)$$

$$\frac{C_a(T)}{10^{19}}\left[\mathrm{cm}^{-3}\right] = 6\left(2 - 3\,e^{-2.1\left(\frac{T}{300\,\mathrm{K}}\right)}\right) \quad (14)$$

$$\frac{C_b(T)}{10^{18}}\left[\mathrm{cm}^{-3}\right] = 6.7 - 12.9\left(\frac{T}{300\,\mathrm{K}}\right)^{0.25} e^{-1.26\left(\frac{T}{300\,\mathrm{K}}\right)} \quad (15)$$

$$\frac{C_3(T)}{10^{16}}\left[\mathrm{cm}^{-3}\right] = 2\left[300 + e^{5.5\left(\frac{T}{300\,\mathrm{K}}\right)}\right] \quad (16)$$

μ_0, α, β, and C_1 in (10) are already defined by (2), (5), (6), and (7), respectively. Again the agreement with experiment is remarkable (Fig. 2). In Fig. 4 the temperature dependence of the minority electron mobility versus the acceptor concentration is shown.

5. Total Electron Mobility

Neglecting dipole interactions between acceptors and donors we finally can write the total electron mobility as:

$$\mu_n^{tot}(N_P, N_{As}, N_B) = \left(\frac{1}{\mu_{n,P,As}(N_P, N_{As})} + \frac{1}{\mu_{n,B}(N_B)} - \frac{1}{\mu_0}\right)^{-1} \quad (17)$$

Note that (17) describes the electron mobility in silicon for an arbitrary doping profile consisting of any combination of P-, As- and B-doping. The extension for Sb is obvious.

6. Conclusion

We have presented a universal electron mobility model for all common dopants in Si. The agreement with experimental data for P- and As-doped Si is remarkably good. For Sb-doped Si further experiments are desired to confirm our results. In case of majorities the mobility formulae are in good agreement with experimental data over the concentration range ($10^{14} - 10^{22}$ cm^{-3}) in the temperature range (70 – 500 K). For minorities we confirm the higher minority mobility compared to the majority values. This model is therefore well suited for device simulation purposes.

References

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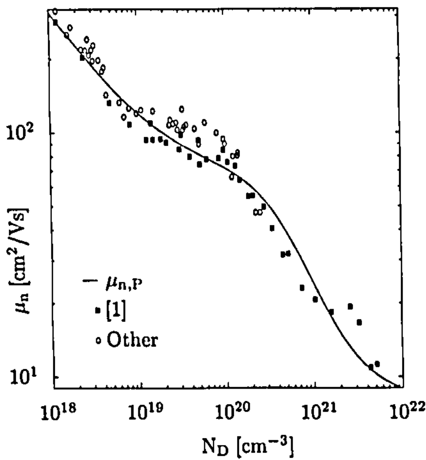

Figure 1: Majority mobility in P-doped Si at room temperature.

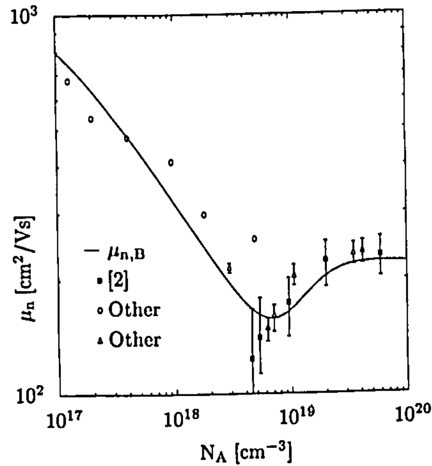

Figure 2: Minority mobility in B-doped Si at room temperature.

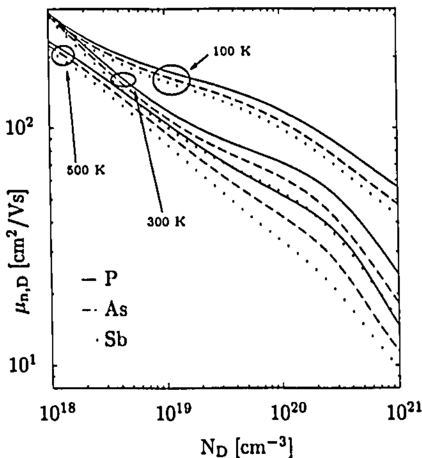

Figure 3: Majority mobility in P-, As-, and Sb-doped Si at 100, 300, and 500 K.

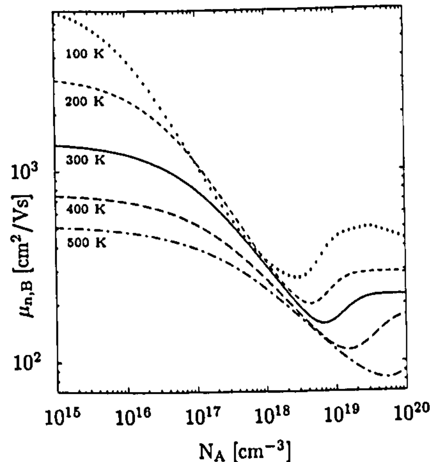

Figure 4: Minority mobility in B-doped Si at various temperatures.